Analysis of The Temperature Data at Hualien between 2010-2020

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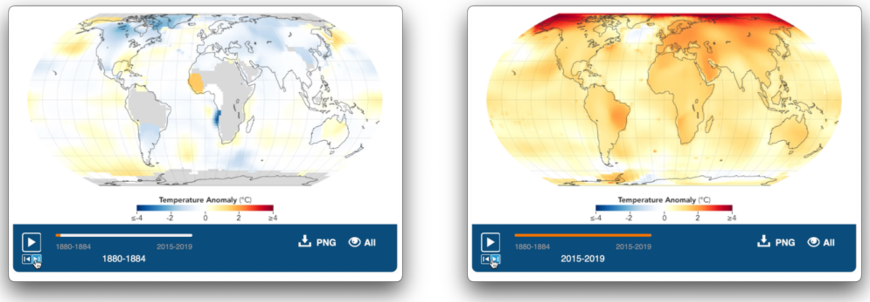
1. Introduction

A. Background

Over the past few decades, scientists have used global temperature, air pressure, precipitation, and more to analyze the causes of climate change. Figure 1 shows how each region changes from the norm. It compares temperatures between 1880-1884/2015-2019 and 1951-1980. It is clear that the data reflect that the climate change after 2015 is more serious than before, and the temperature has risen by nearly 2-4 degrees.



▲ Figure 1

B. Goal

Analysis of ocean and air temperature in Hualien from 2010 to 2020 using probability distribution, Fourier transform, and Hilbert-Huang transform. Also, find out the physical meaning of the analysis results.

C. CWB Observation Data Inquire System

I get data from CWB Observation Data Query System. The sampling frequency is 1 hour, and the data from Hualien buoys from July 2010 to June 2020 are selected for analysis.

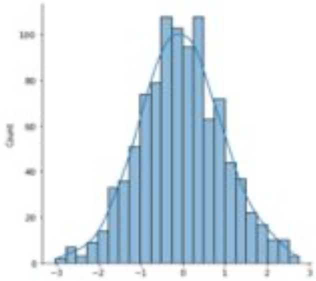
2. Analysis method

A. Probability distribution

A simple way to find the probability distribution of data. First, get a time series data. Statistics and draw histograms. Finally, Gaussian distribution or normal distribution is fitted to the histogram as follows to obtain the equation,

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where symmetry with μ, width σ, height $\frac{1}{\sigma\sqrt{2\pi}}$.

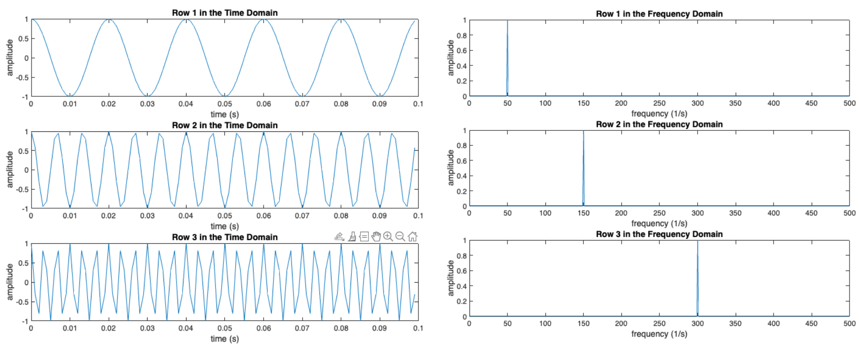


B. Fourier spectral analysis

Transform the time series data from time domain to frequency domain by the following equation:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt.$$

Take $\cos(2\pi * 50t)$, $\cos(2\pi * 150t)$, $\cos(2\pi * 300t)$ for example. After taking the Fourier transform, we can see there are contribution at $50\ \mathrm{Hz}$, $150\ \mathrm{Hz}$, $300\ \mathrm{Hz}$ on the frequency domain, respectively.



C. Hilbert-Huang transform

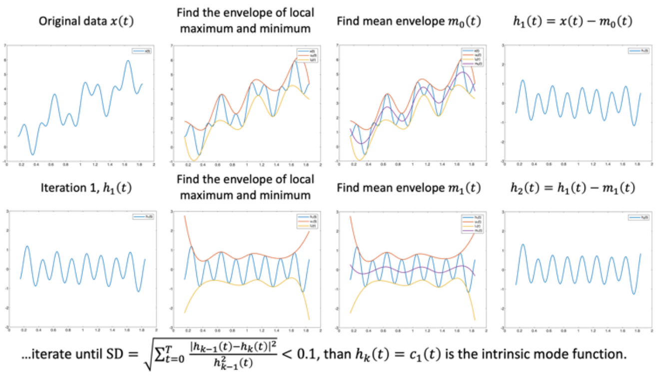
Decompose the signal into so-called intrinsic mode functions and apply the Hilbert transform to each function. Take $x(t) = 0.7\sin(2\pi * 5t) + \sin(2\pi * 2t) + 5\sin(2\pi * 0.1t)$ for example.

i. Use empirical mode decomposition (EMD) process to decompose the data into intrinsic mode functions. The intrinsic mode functions (IMF) c_j has two properties:

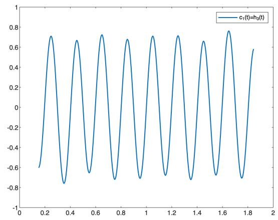
- In the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ at most by one.
- At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima

is zero.

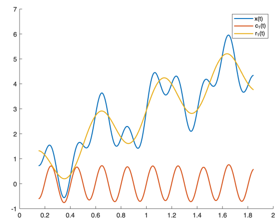
The process of EMD shows in the following figure.



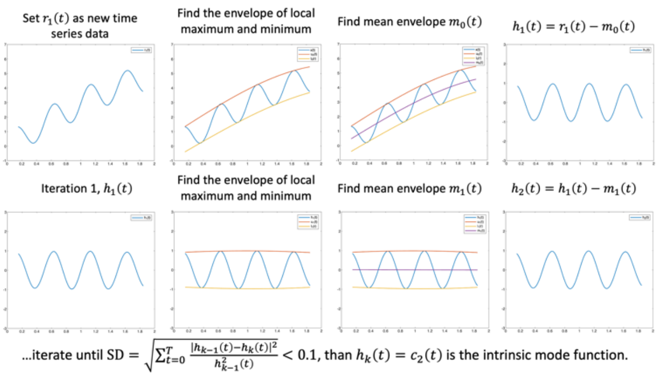
In this example, $h_3(t) = c_1(t)$ is IMF1.



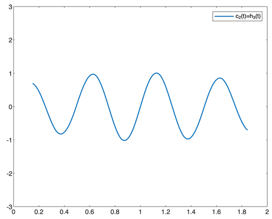
Next, subtract IMF1 $c_1(t)$ from the original data $x(t)$ to get $r_1(t)$. This step subtracts the high frequencies from the data, leaving low frequency signals.



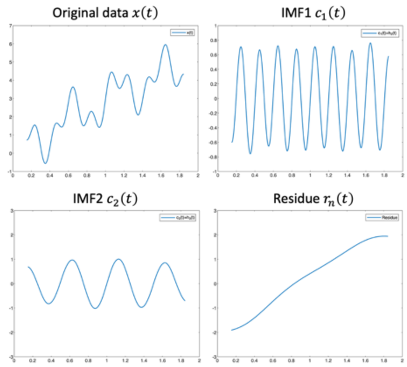
View $r_1(t)$ as the data, do the EMD process again.



In this example, $h_2(t) = c_2(t)$ is IMF2.



Finally, we can express our data as $x(t) = \sum_{j=1}^{n} c_j + r_n$. $r_n(t)$ is the residue function.

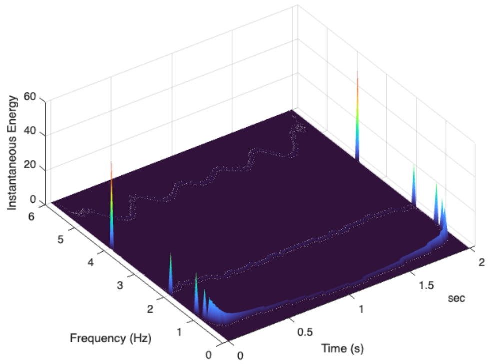


ii. Do the Hilbert transform at each IMF

$$a_j(t) = c_j(t)e^{-i\omega_j(t)}, \omega_j(t) = \frac{d\theta_j}{dt}.$$

iii. Compose each $a_j(t)$ component by

$$x(t) = \Re \sum_j a_j(t) e^{i\int_t \omega_j(\tau)d\tau}.$$

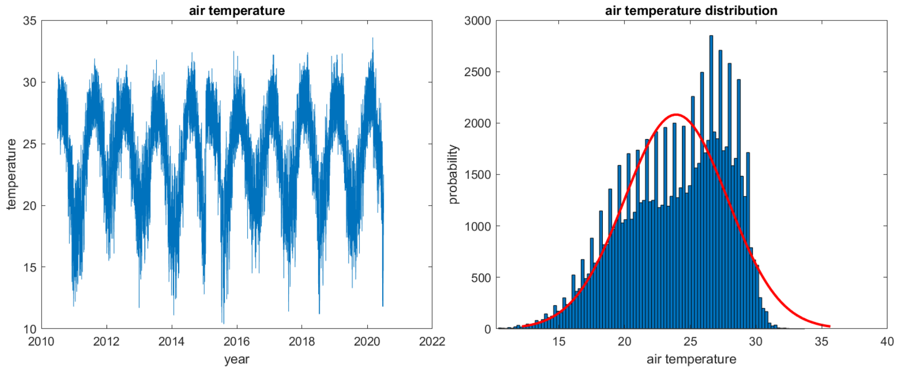


Finally, we can see three bright bands in the time-frequency space. The places where the bright bands appear correspond to the frequencies we set at the beginning, which are 0.1 Hz, 2 Hz, and 5 Hz.

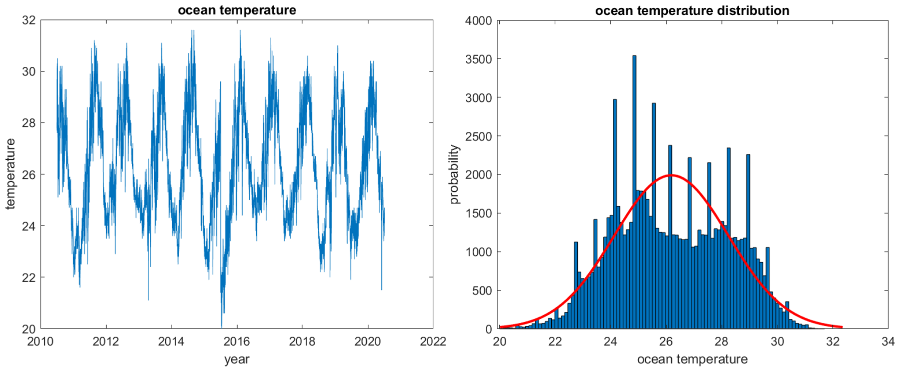
3. Analysis result

A. Probability distribution of temperature

Figure 2 shows the air temperature data and air temperature histogram. The average temperature is $\mu_a = 23.9219$℃, and the variance is $\sigma_a = 3.91225$℃. Figure 3 shows the ocean temperature data and ocean temperature histogram. The mean value of ocean temperature is $\mu_o = 26.1828°\text{C}$, and the variance is $\sigma_o = 2.05571°\text{C}$.



▲ Figure 2



▲ Figure 3

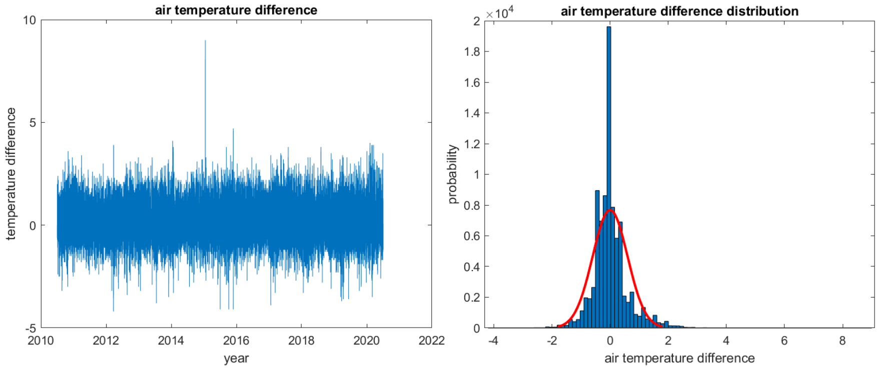
Through the results of air and sea, I discovered some of the above properties:

a. According to the latest data from the National Oceanic and Atmospheric Administration (NOAA), the global average ocean surface temperature since April is $21.1°\text{C}$. However, the average temperature in the waters of Hualien in the past ten years is $26.1828°\text{C}$, which is higher than the global average. I think it's because the Kuroshio Current flows through Taiwan, pumping warm sea water into Taiwan.

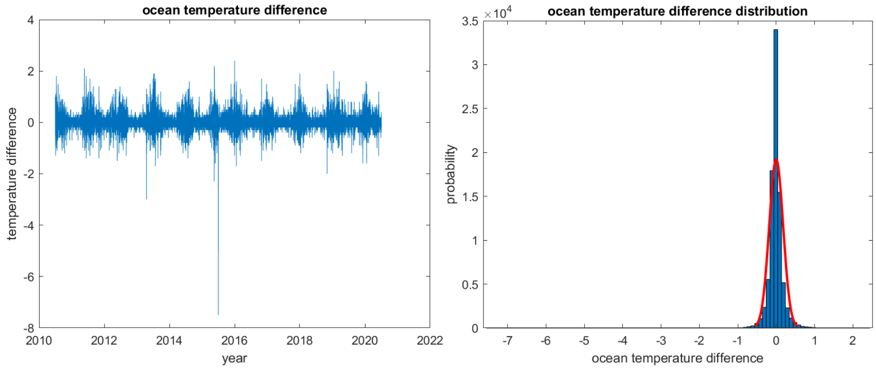
b. Variance indicates how spread out the data is. Comparing the variance of ocean temperature and air temperature, $\sigma_a > \sigma_o$ indicates that the air temperature varies greatly. The reason is that water has a higher specific heat than air.

B. Probability distribution of temperature difference

Figure 4 is the air temperature difference data and the temperature difference histogram. The average value of the air temperature difference is $\mu_{da} = -8.9 \times 10^{-5}°\text{C}$ and the variance is $\sigma_{da} = 0.60126°\text{C}$. Figure 5 is the data of ocean temperature difference and histogram of ocean temperature difference. The average ocean temperature difference is $\mu_{do} = -4.5 \times 10^{-5}$°C, and the variance is $\sigma_{do} = 0.18221$°C.



▲ Figure 4



▲ Figure 5

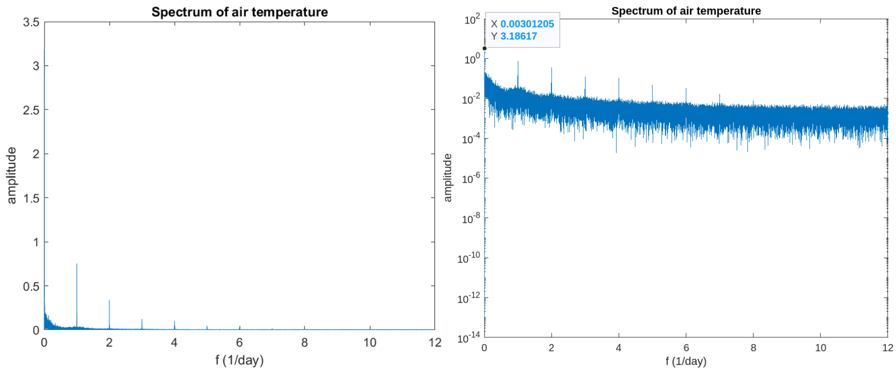
Going through the air and sea results, I found some of the properties above:

a. $|\mu_{da}| > |\mu_{do}|$ and $\sigma_{da} > \sigma_{do}$ all indicate that the air temperature has a large temperature change, and the specific heat of water is greater than that of air.

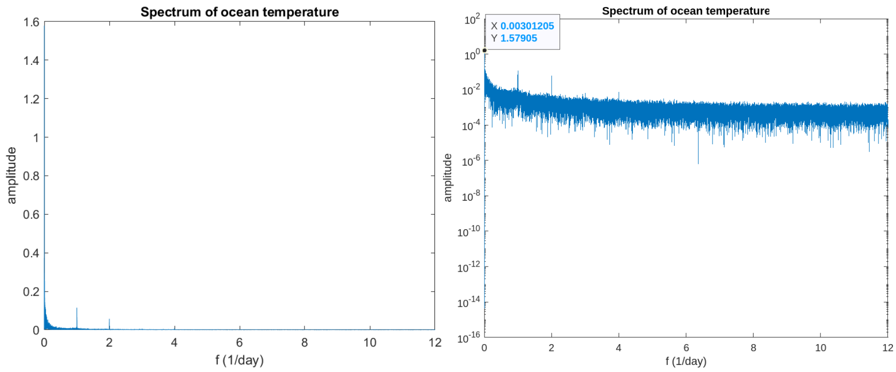
b. I use data for stability analysis. Through Lyapunov exponent, $\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln|f'(x_i)|$. For stable or period system, λ is negative. However, when I calculate the exponent of ocean temperature and air temperature, they both are $-\infty$, which means that both systems are very stable. Therefore, I ignore the value of $f'(x_i) = 0$ and re-find the exponent. The exponent of air temperature is -1.0273 and the exponent of ocean temperature is -1.1751, indicating that the ocean temperatures is more stable than the air temperatures.

C. Fourier spectral analysis

Figure 6 and Figure 7 show the Fourier spectra of air and ocean temperature. The figures on the right are plotted by logarithmic scale on y-axis.



▲ Figure 6



▲ Figure 7

Through the results of Fourier transform, I concluded the following phenomena:

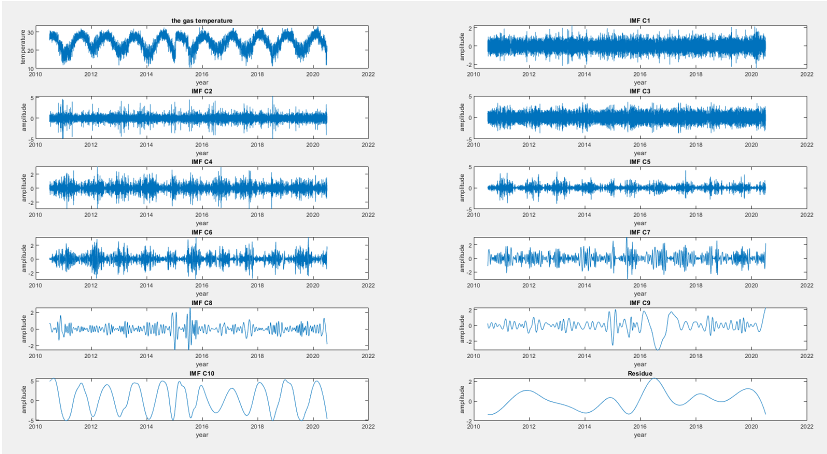
a. Both ocean temperature and air temperature have a main frequency, 0.003 times per day, which corresponds to 1 time per year. The reason is because of the revolution of the earth.

b. Due to diurnal variation, the secondary frequency is 1 time per day. In addition, the change of sea and land breeze is also once a day.

c. It can be seen from the spectrum that there is some high frequency modulation. For example, 2 times a day is equivalent to half of the contribution of 1 time a day. My understanding is that the land breeze and sea breeze have a certain lag in the adjustment of the temperature, and the temperature change should be minimized to make the temperature system reach a balance. However, the air temperature system has more high frequency modulations than the ocean temperature. It can be seen from Figure 4 that the change in air temperature is more drastic. Therefore, air requires more high frequency terms to modulate the system.

D. Hilbert-Huang transform

Figure 8 and Figure 9 show the raw data, IMFs, and residue for the air temperature and ocean temperature. In the 10^{th} IMF in the two graphs, there are 10 shocks in ten years because of the earth's revolution.



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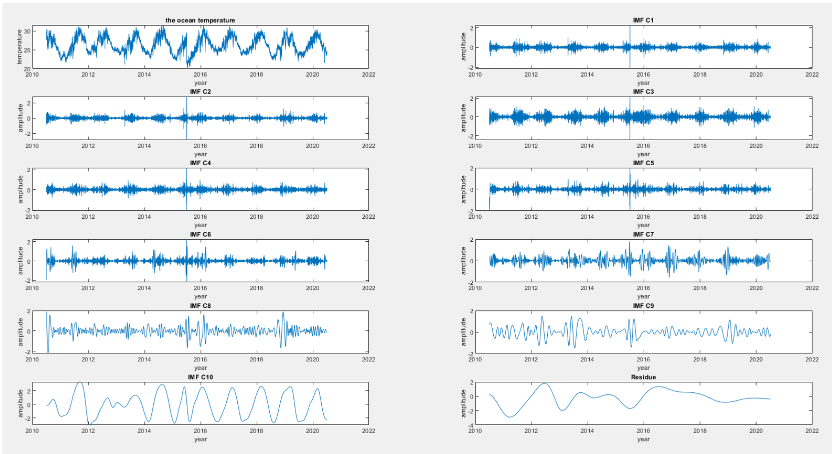
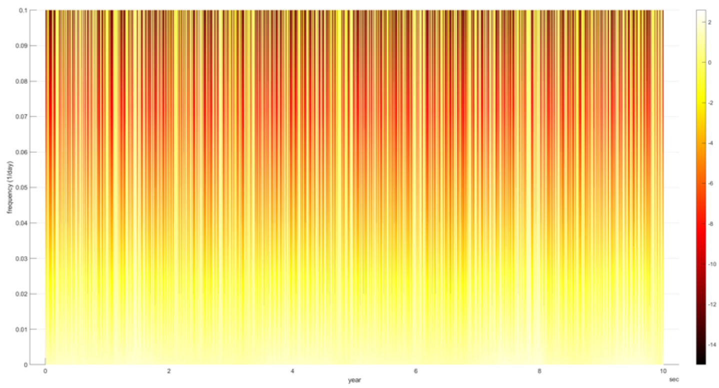


▲ Figure 9

Figure 10 and Figure 11 show the Hilbert-Huang spectra plotted on a logarithmic scale on the y-axis and limited to $[0,0.1]$ per day. The brightest band is around $0\sim0.1$ times per day, which is consistent with the results of IMF and Fourier transform.



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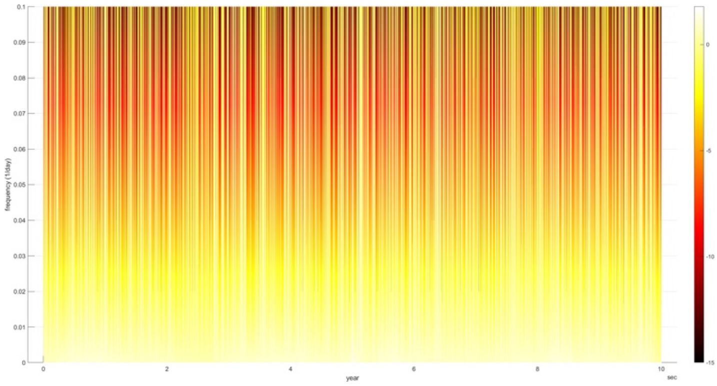

▲ Figure 11

4. Conclusion

A. By statistic method:

- $\mu_o > 21.1$℃: due to the Kuroshio current passing through Taiwan
- $|\mu_{da}| > |\mu_{do}|$ and $\sigma_a > \sigma_o$: the specific heat of water is greater than that of air.

B. Stability Analysis with Lyapunov exponent:

- The λ of both air temperature and ocean temperature is $-\infty$, which means the systems are very stable.
- Excluding $f'(x) = 0$, the exponent of air temperature is greater than the exponent of ocean temperature. This means that the ocean temperatures are more stable than the air temperatures. The physical meaning is that the specific heat of water is higher than that of air.

C. By Fourier transform and Hilbert-Huang transform:

- Both temperature data are on a daily basis: corresponding to the diurnal variation as well as the land and sea breeze.
- Both temperature data have a once-a-year cycle: corresponding to the revolution of the earth.

5. Reference

A. https://earthobservatory.nasa.gov/world-of-change/global-temperatures

B. https://ocean.cwb.gov.tw/V2/

C. https://tw.stock.yahoo.com/news/海洋表面溫度再創新高-專家發警告-124947018.html