One-Dimensional Simulation with the FDTD Method

2020.10.20

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Outline

- One-Dimensional free space formulation
- Boundary condition
  - Fixed Boundary condition
  - Absorb boundary condition
- Propagation Medium
  - Dielectric Medium
  - Lossy Dielectric Medium

One-Dimensional free space formulation

Maxwell's equation (3D)

$$\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times \boldsymbol{H} - \mathbf{J}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \boldsymbol{E}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \boldsymbol{H} = 0$$

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Use to decide initial condition.

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$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_0}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)$$

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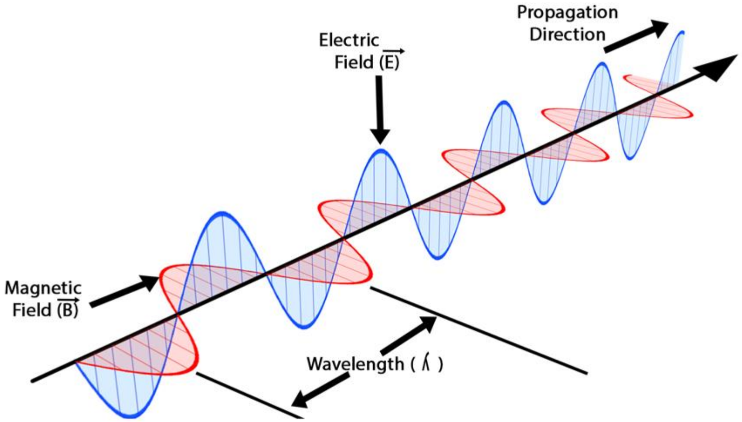
E_x & H_y, traveling in the z direction.



One-Dimensional free space formulation

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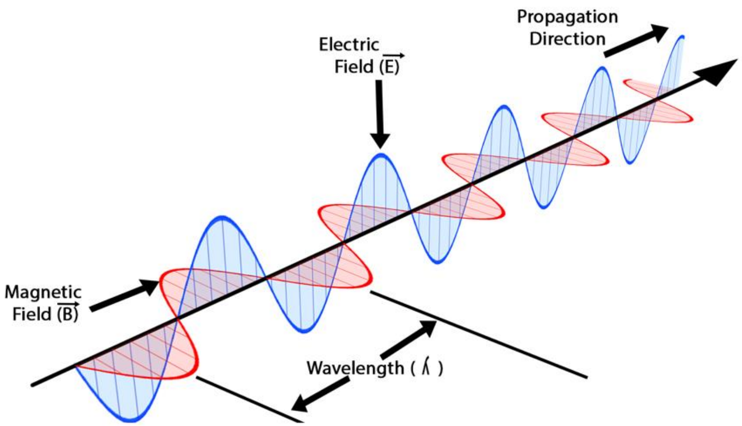
$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \boldsymbol{E}$$

Simplify as 1D

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0} \frac{\partial H_y}{\partial z}$$
$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z}$$

E_x & H_y, traveling in the z direction.



One-Dimensional free space formulation

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$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}$$

$\downarrow$ Use central-difference formula

$$\frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H_y^n\left(k+\frac{1}{2}\right) - H_y^n(k-\frac{1}{2})}{\Delta z}$$
$$\frac{H_y^{n+1}\left(k+\frac{1}{2}\right) - H_y^n(k+\frac{1}{2})}{\Delta t} = -\frac{1}{\mu_0}\frac{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n-\frac{1}{2}}(k)}{\Delta z}$$

$\downarrow$

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) - \frac{dt}{\varepsilon_0 \cdot dz}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$
$$H_y^{n+1}\left(k+\frac{1}{2}\right) = H_y^n(k+\frac{1}{2}) - \frac{dt}{\mu_0 \cdot dz}\left[E_x^{n+\frac{1}{2}}(k+1) - E_x^{n-\frac{1}{2}}(k)\right]$$

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$\downarrow$

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) - \frac{dt}{\varepsilon_0 \cdot dz}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$
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How to iteration?

One-Dimensional free space formulation

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) - \frac{dt}{\varepsilon_0 \cdot dz}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$

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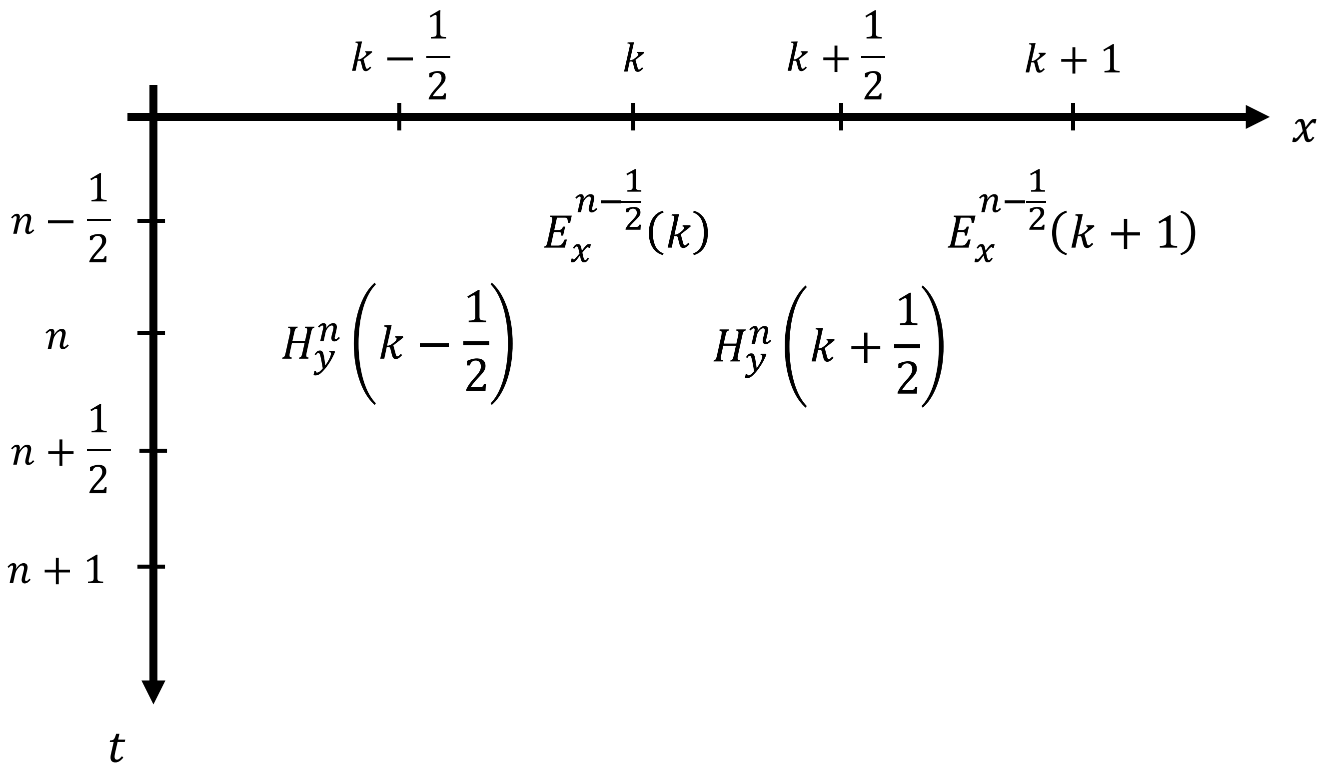

One-Dimensional free space formulation

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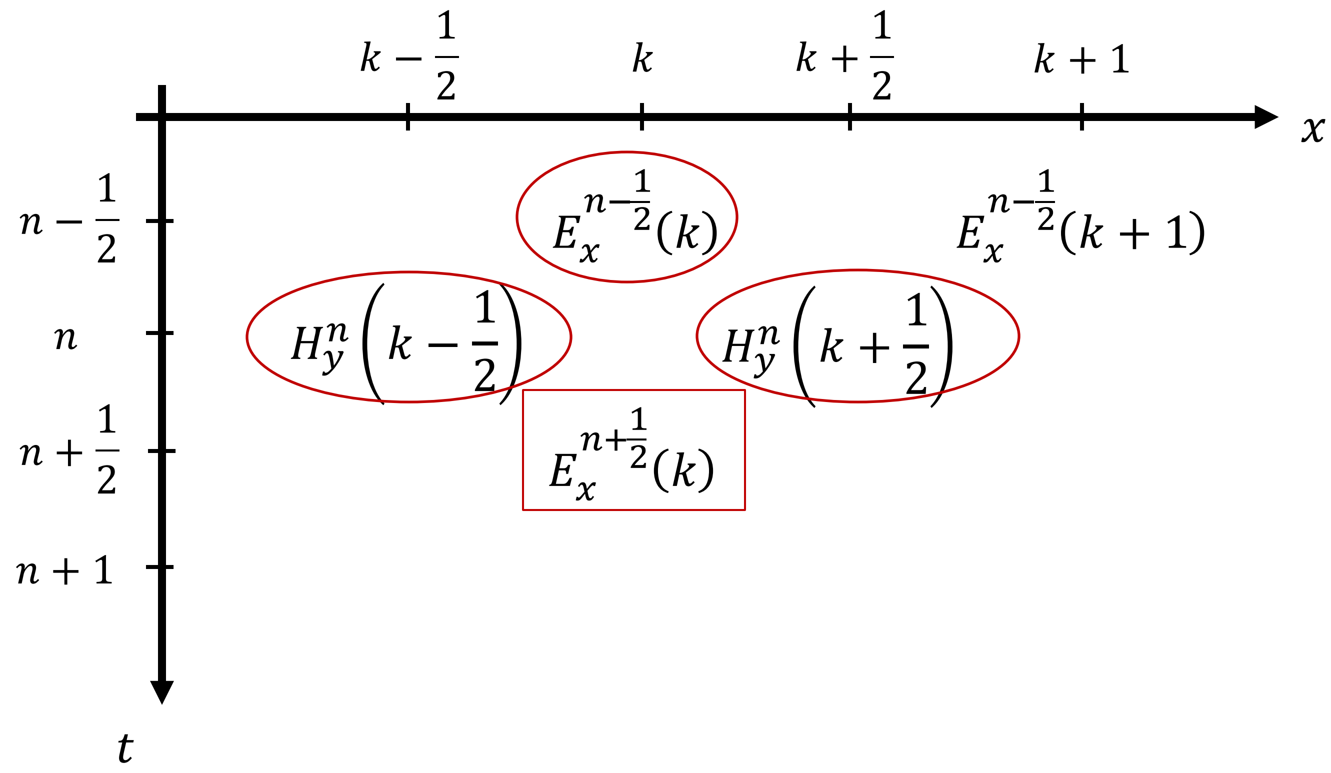

One-Dimensional free space formulation

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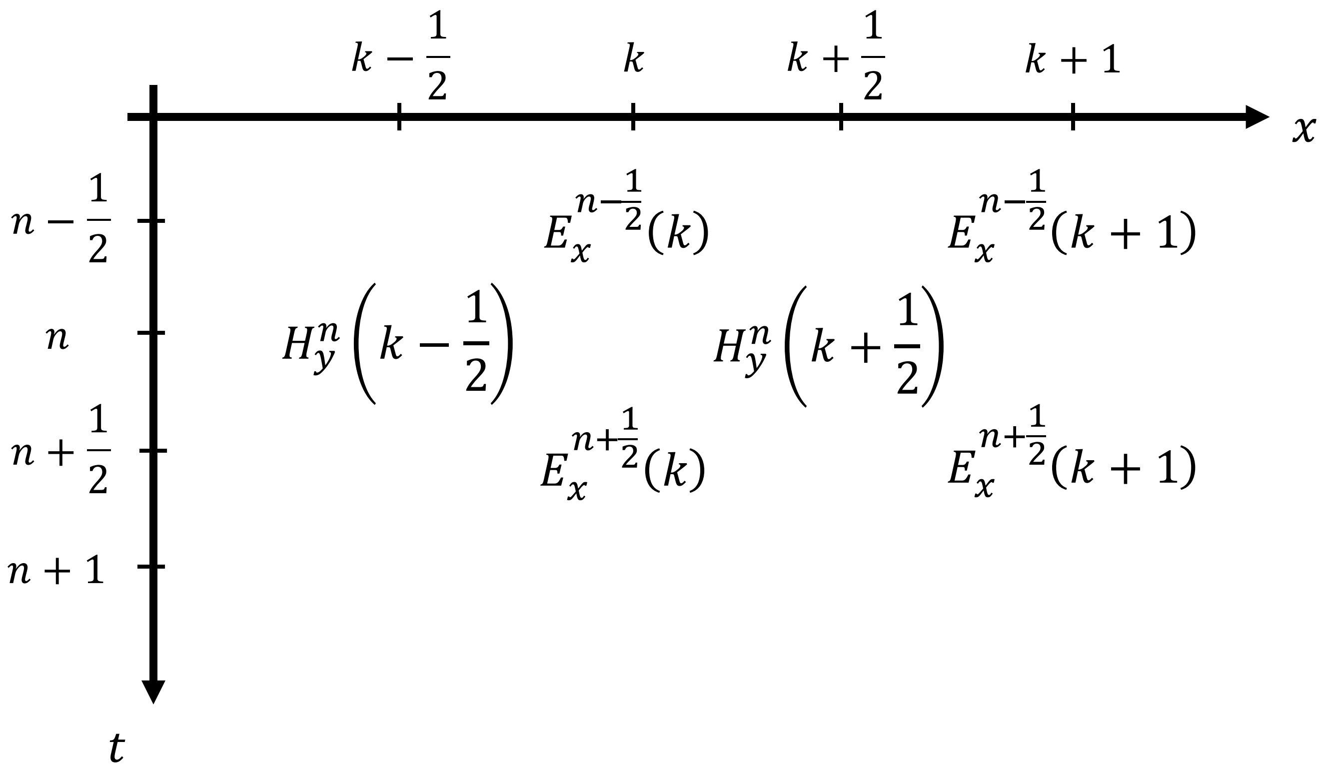

Except the last position of E_x

One-Dimensional free space formulation

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) - \frac{dt}{\varepsilon_0 \cdot dz}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$

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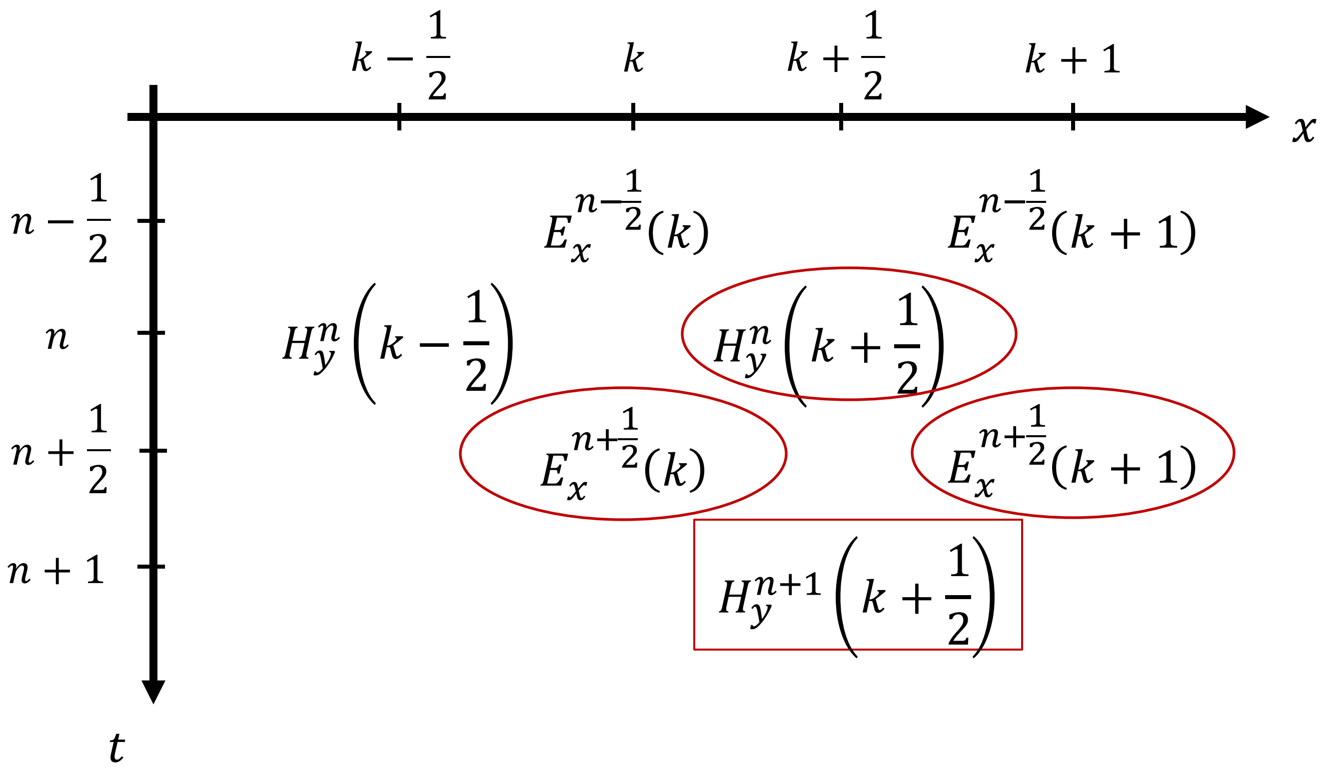

Except the last position of E_x

One-Dimensional free space formulation

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) - \frac{dt}{\varepsilon_0 \cdot dz}\left[H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)\right]$$

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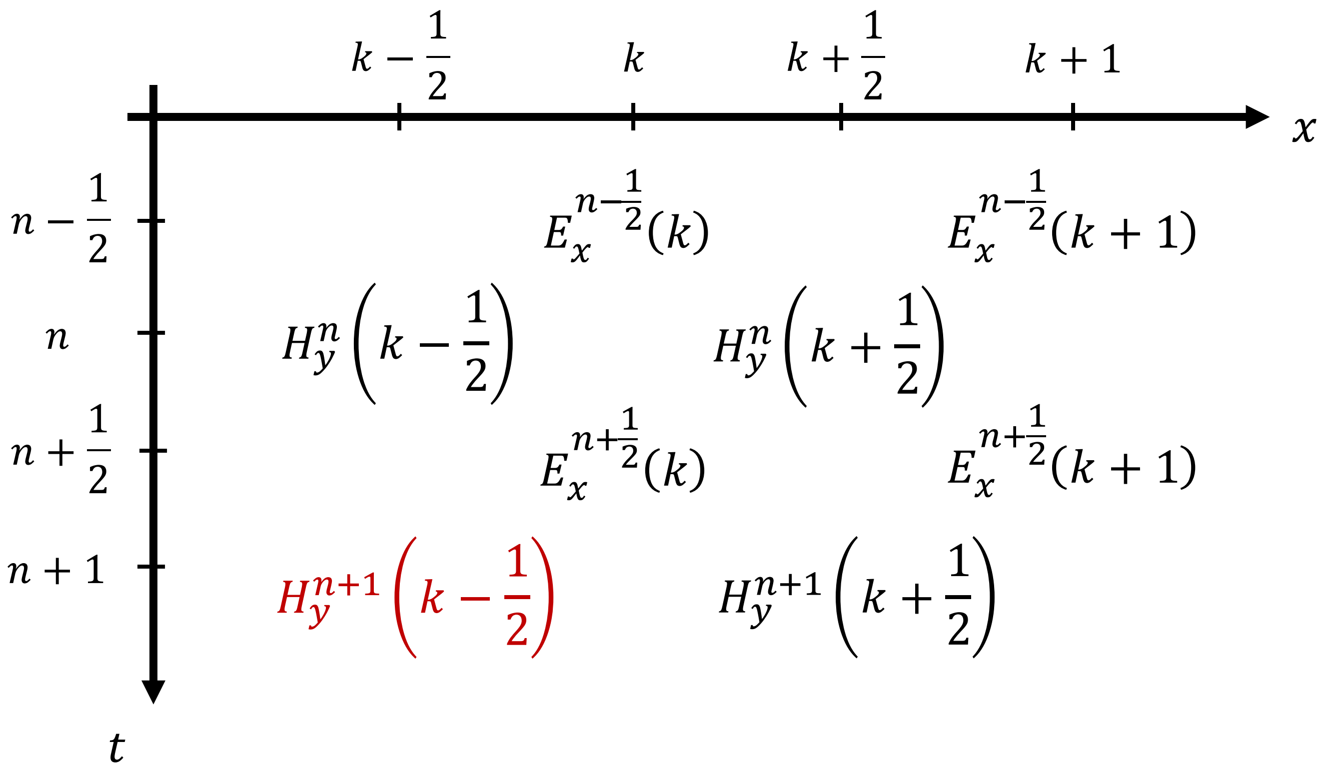

Except the last position of E_x and the first position of H_y.