Mechanical energy

$$E = T + V = \sum_i \frac{1}{2} m_i v_i^2 + m_i g h_i$$

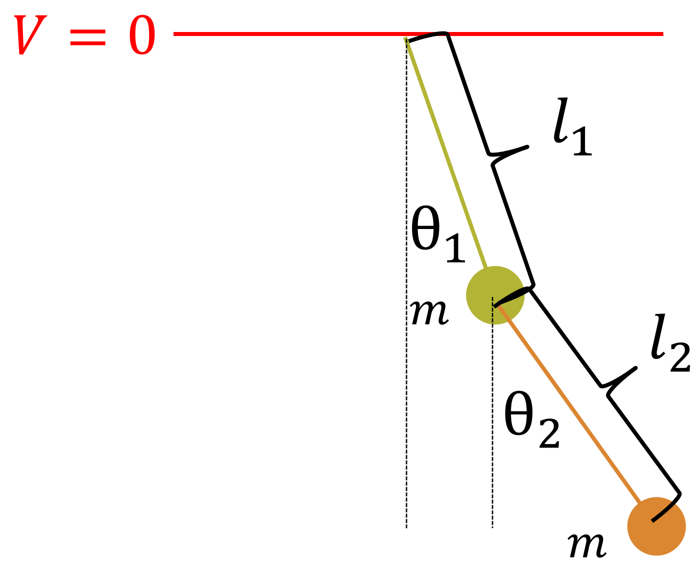
Before releasing pendulums, kinetic energy is 0.

Total energy of system: $E = V_0$

i: pendulum V_0: initial potential energy

Energy of the pendulum

Potential energy of the system:



$$E = V_0 = -m_1 g \frac{1}{2} l_1 \cos\theta_1 - m_2 g \left(l_1 \cos\theta_1 + \frac{1}{2} l_2 \cos\theta_2 \right)$$
$$= -\frac{1}{2} mg(3l_1 \cos\theta_1 + l_2 \cos\theta_2)$$

Energy of the system

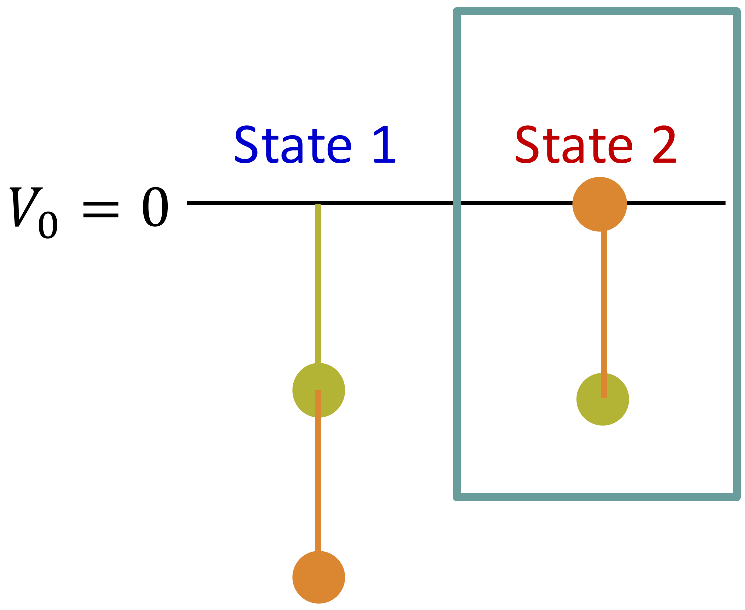
$$E = -\frac{1}{2}mg(3l_1\cos\theta_1 - l_2\cos\theta_2)$$

Energy of state 1.

$$E_1 = -\frac{1}{2}mg(3l_1 + l_2)$$

Energy of state 2.

$$E_2 = -\frac{1}{2}mg(3l_1 - l_2)$$



If $E > E_2$, energy used to flip is enough.

Bifurcation

Motion changes when energy is beyond a threshold.

$$E = E_2$$

$$E < E_2$$

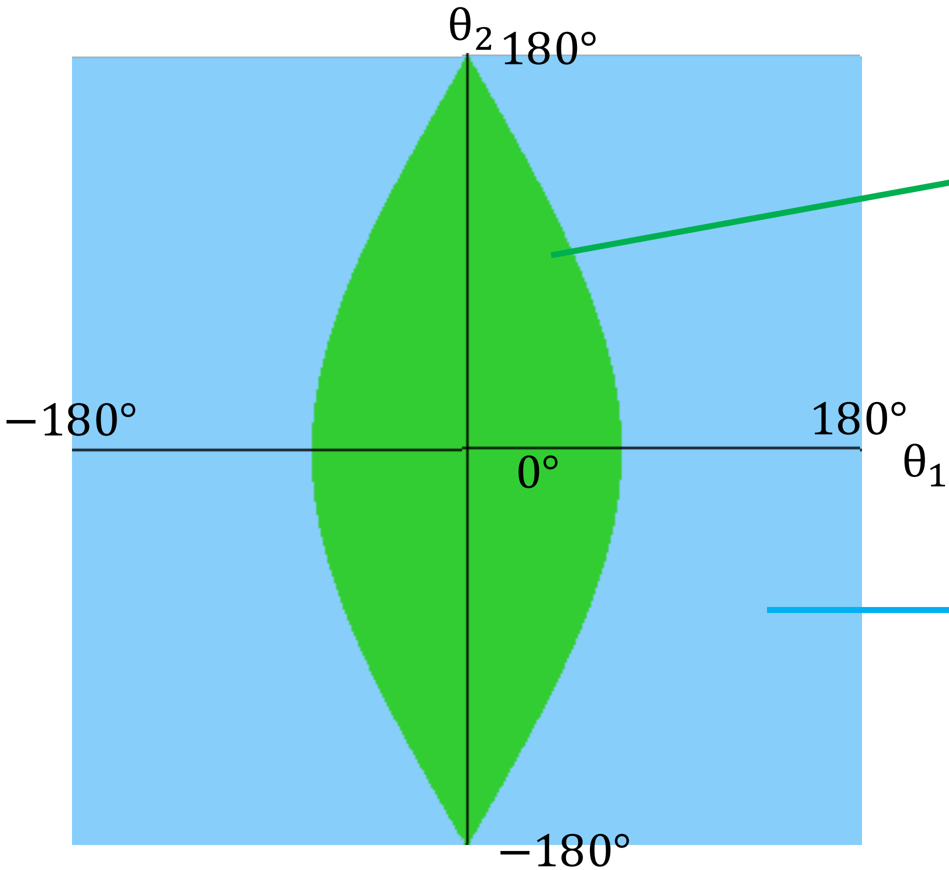
Single-pendulum-like motion.

$$E > E_2$$

Chaotic motion.

Energy of the pendulum

By comparing E and E_2



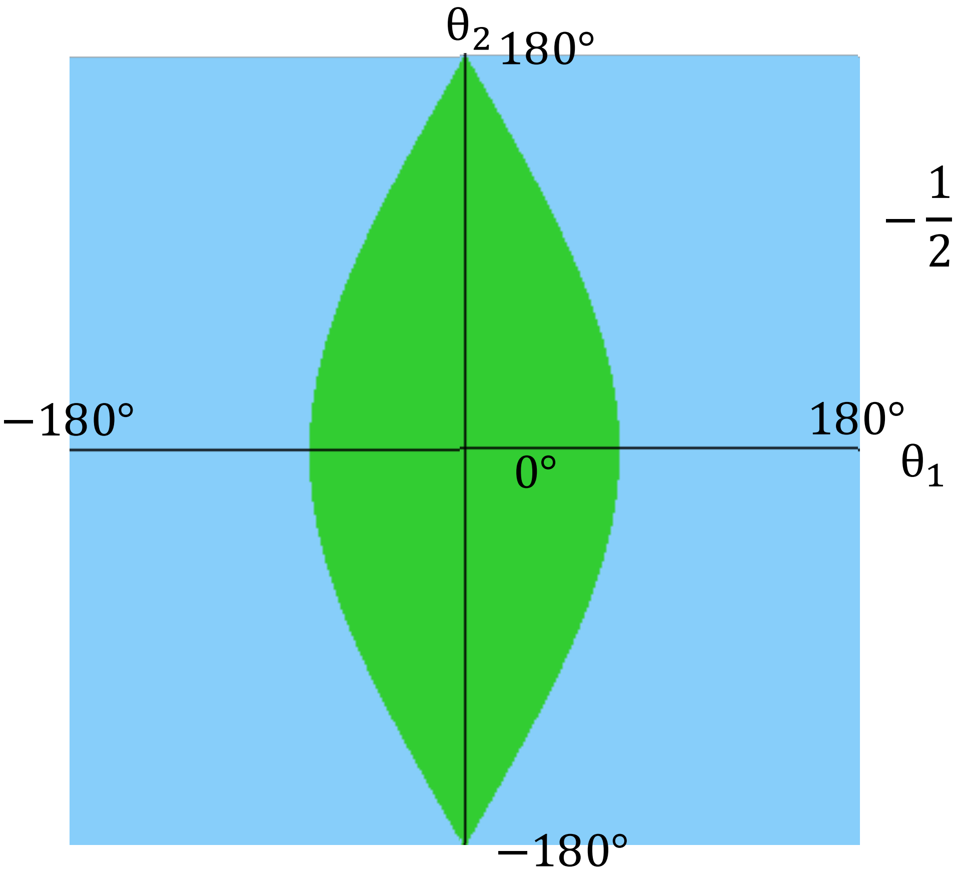
$E < E_2$:
Lower pendulum cannot flip

$E > E_2$:
Lower pendulum flip

$m = 10\ (kg)\ \ l_1 = 1\ (m)\ \ \ l_2 = 1\ (m)$

Energy of the pendulum

By comparing E and E_2



$$E > E_2$$

$$-\frac{1}{2}m(3l_1\,cos\,\theta_1 + l_2\,cos\,\theta_2) > -\frac{1}{2}m(3l_1 - l_2)$$

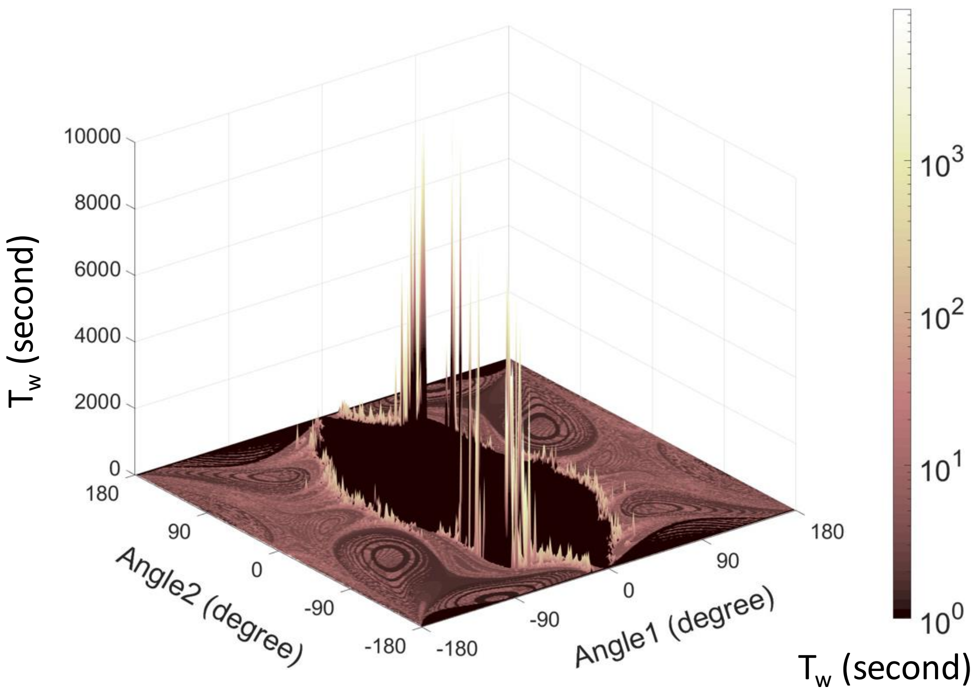
In this case

$$3\,cos\,\theta_1 + cos\,\theta_2 < 2$$

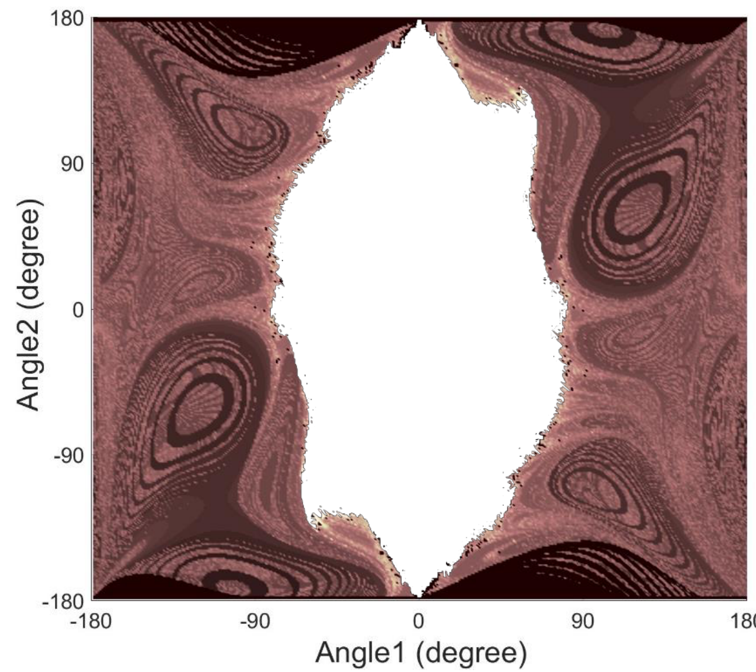
$m = 10\,(kg)$ $l_1 = 1\,(m)$ $l_2 = 1\,(m)$

Phase diagram

Detecting the angle of lower pendulum

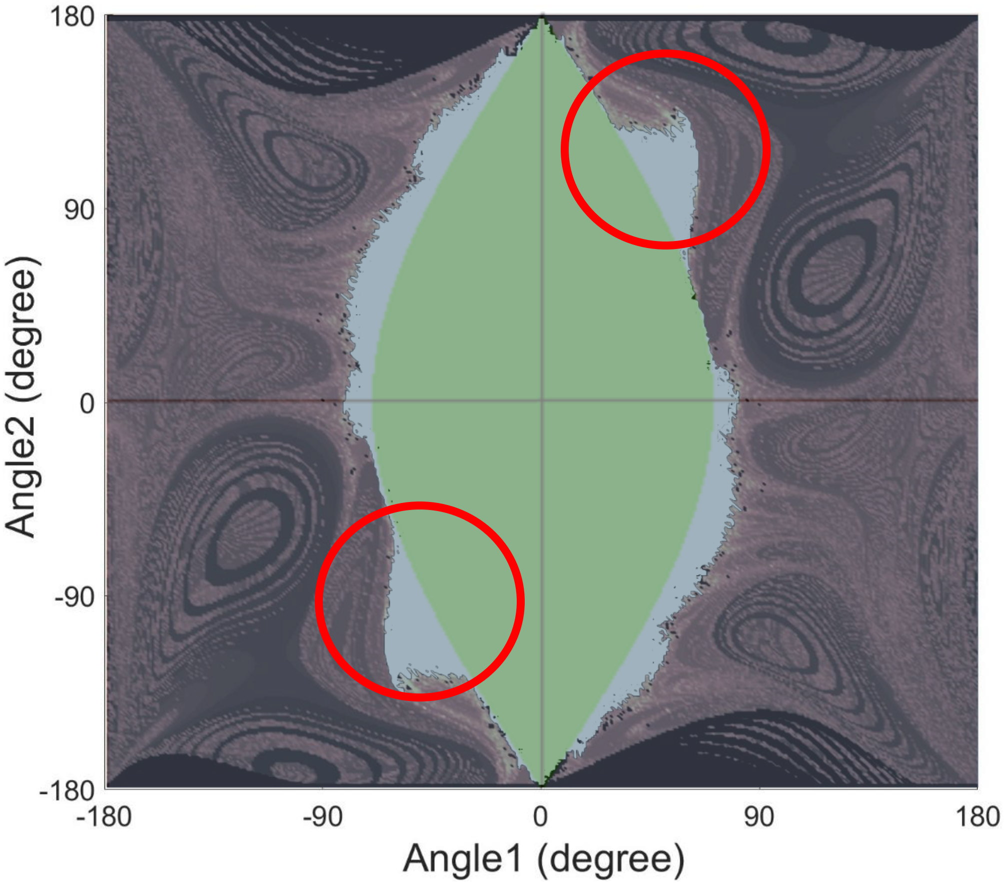


Top view



Observe time: 10000 (s) T_w: Waiting time before first flipping
Initial angle: $\theta_1 = -180° \sim 180°$ $\theta_2 = -180° \sim 180°$ Delay time: 0.05 (s)

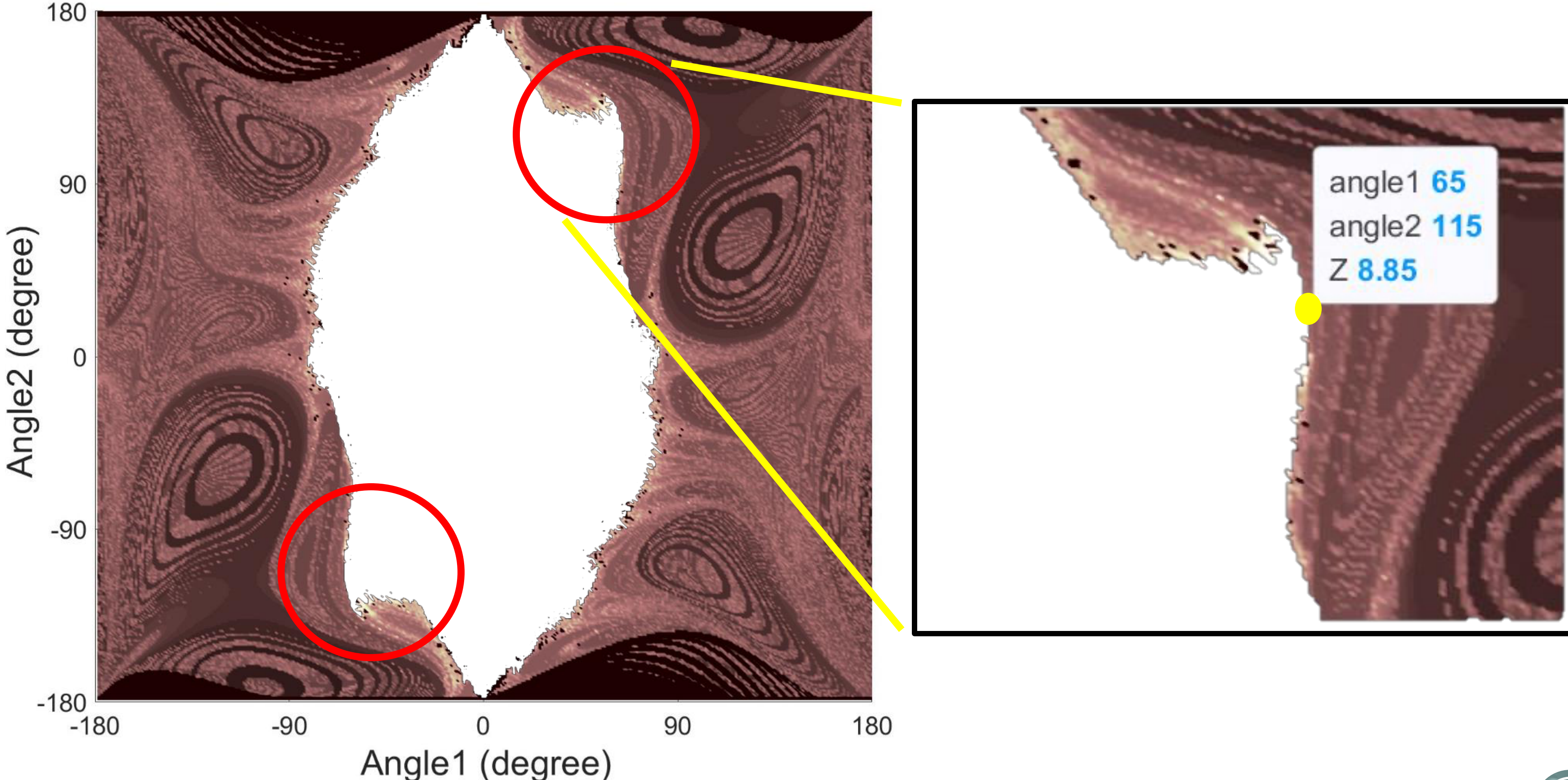
Compare with the energy result



Why there are two empty regions?

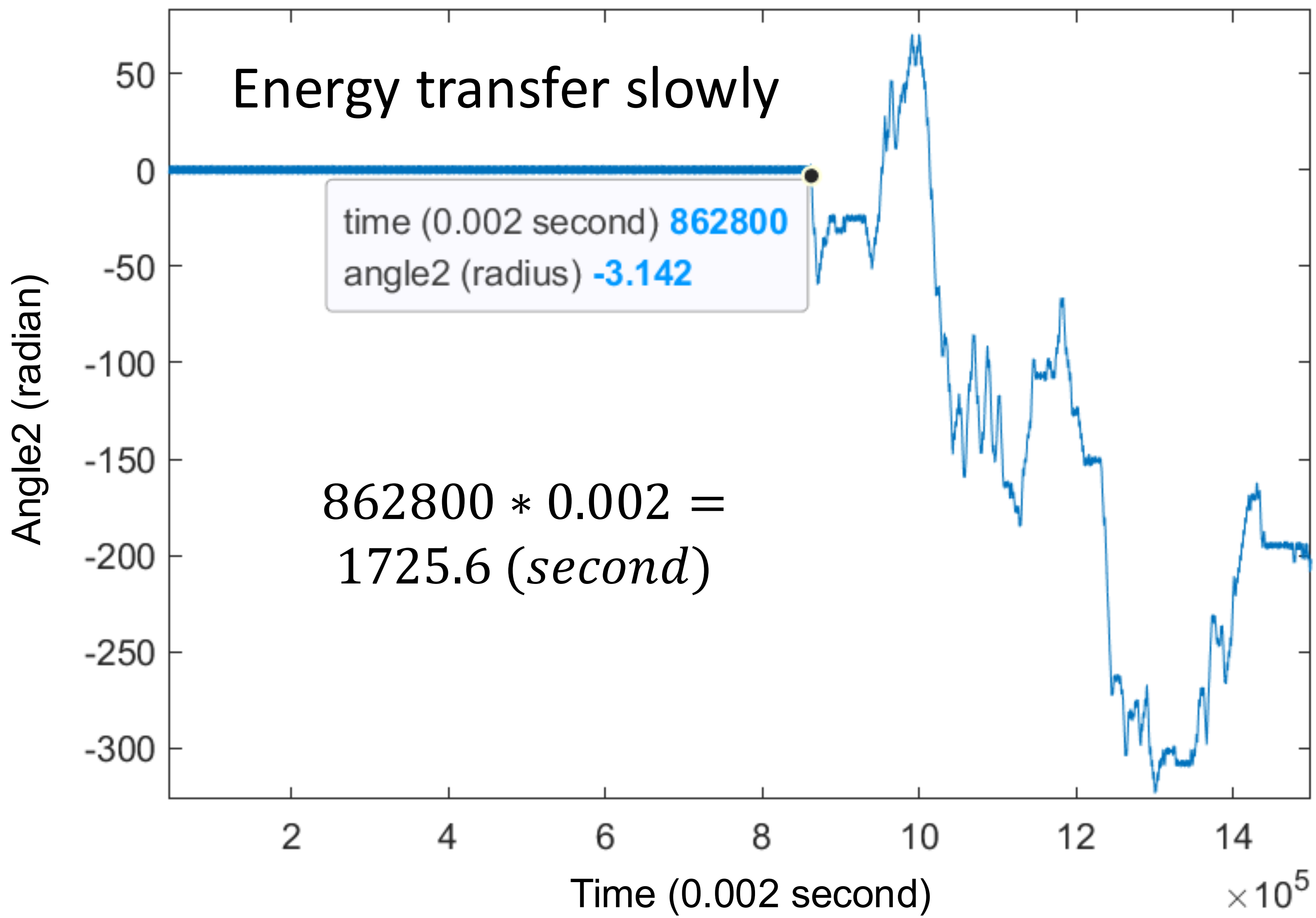
Phase diagram

We focus on the edge of flipping

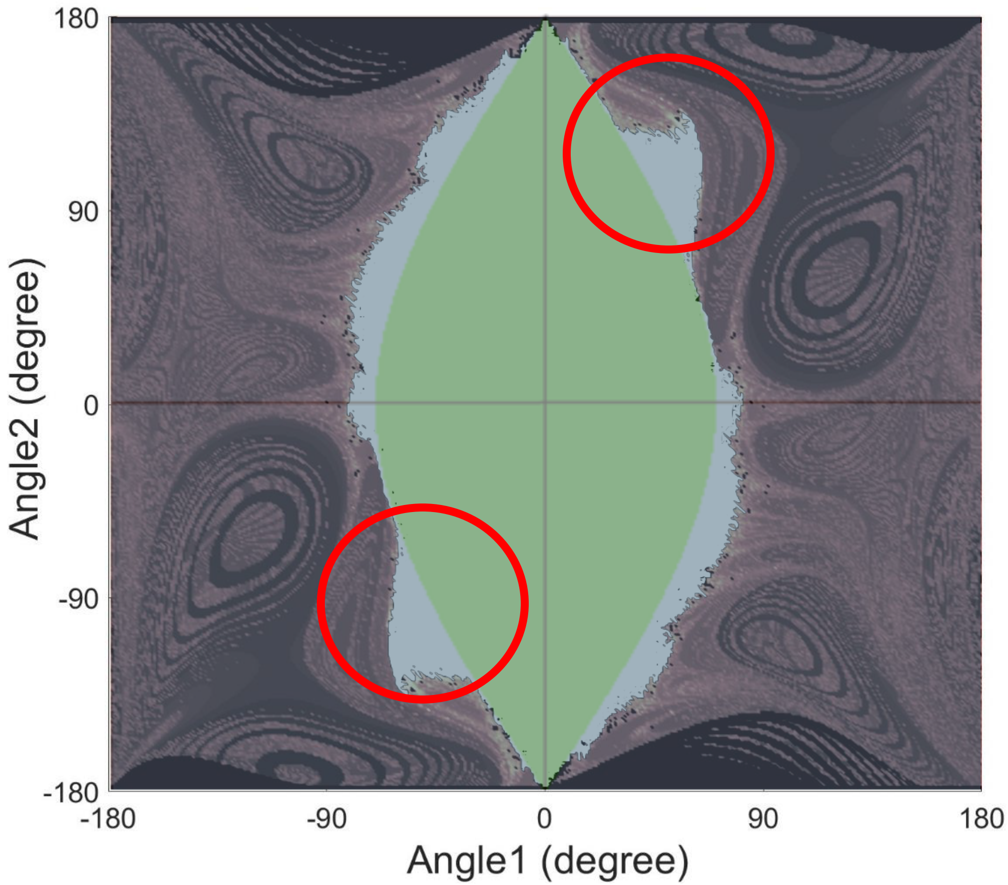


Time series

Initial condition
(64.336, 115)



States nearby empty regime



Nearby stable states

Taking lots of time to flip

Why?

Flipping with long waiting time

Condition of flipping:

$$\theta_1 \approx 0, \omega_1 \approx 0$$

Low total energy

➡ Few states can flip.

$(62°, 60°)$