Work report

2022.03.16

Normalization

Fundamental parameter:

$$\begin{cases} \tilde{t} = \dfrac{t}{\dfrac{1}{\omega_p}} = \omega_p t \\ \tilde{x} = \dfrac{x}{\lambda_D} \\ \tilde{q} = \dfrac{q}{e} \\ \tilde{m} = \dfrac{m}{m_e} \\ \tilde{\epsilon} = \dfrac{\epsilon}{\epsilon_0} \end{cases}$$

$$\longrightarrow$$

Derived parameter:

$$\begin{cases} \tilde{n} = \dfrac{n}{n_0} = \dfrac{n}{\dfrac{\omega_p^2 m_e \epsilon_0}{e^2}} \\ \tilde{v} = \dfrac{v}{\lambda_D \omega_p} \\ \tilde{\rho} = \dfrac{\rho}{e n_0} \\ \tilde{\phi} = \dfrac{\phi}{\dfrac{\lambda_D^2 e n_0}{\epsilon_0}} \\ \tilde{E} = \dfrac{E}{\dfrac{\lambda_D e n_0}{\epsilon_0}} \end{cases}$$

Energy and momentum:

$$\begin{cases} \tilde{K} = \dfrac{K}{\dfrac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}} \\ \tilde{U} = \dfrac{U}{\dfrac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}} \\ \tilde{p} = \dfrac{p}{\dfrac{\lambda_D^2 \omega_p^3 m_e^2 \epsilon_0}{e^2}} \end{cases}$$

```
omega_D = 1
Debye_length = 1
elementary_charge = 1.602 * 1e-19
elementary_mass = 9.109 * 1e-31
epsilon0 = 8.854 * 1e-12

n0 = omega_D * omega_D * elementary_mass * epsilon0 / elementary_charge / elementary_charge
v_nor = Debye_length * omega_D
rho_nor = elementary_charge * n0
phi_nor = Debye_length * Debye_length * n0 * elementary_charge / epsilon0
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energy_nor =  Debye_length **3 * omega_D **4 * elementary_mass **2 * epsilon0 / elementary_charge **2
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Define one and the other can be determined.

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Real particle density

Energy and momentum:

$$\begin{cases} \tilde{K} = \dfrac{K}{\frac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}} \\ \tilde{U} = \dfrac{U}{\frac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}} \\ \tilde{p} = \dfrac{p}{\frac{\lambda_D^2 \omega_p^3 m_e^2 \epsilon_0}{e^2}} \end{cases}$$

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```

Normalization

Real particle number per grid: $N_{real} = n_0 * \Delta x$

Real particle number per macroparticle: $w = \frac{N_{real}}{ppc} = \frac{N_{real}}{\frac{NP}{NG}} = \frac{n_0 * \Delta x * NG}{NP}$

$\downarrow$

$$\widetilde{N}_{real} = \tilde{n}_0 * \Delta\tilde{x} = \frac{n_0}{n_0} * \frac{\Delta x}{\lambda_D} = \frac{N_{real}}{n_0 \lambda_D}$$

$$\widetilde{w} = \frac{\tilde{n}_0 * \Delta\tilde{x} * NG}{NP} = \frac{n_0}{n_0} * \frac{\Delta x}{\lambda_D} * \frac{NG}{NP} = \frac{w}{n_0 \lambda_D}$$

Normalization

Charge per macroparticle: $q_{macro} = q * w$

(NOTICE!! In 1D PIC code, the SI unit of q and m are $\frac{C}{m^2}$ and $\frac{kg}{m^2}$.)

Macroparticle density:

$$\rho = q_p \sum_i \delta(x_i) = \frac{qw}{\Delta x} = q n_0 \frac{NG}{NP} = \frac{q N_{real}}{ppc * \Delta x} = \frac{q n_0}{ppc} \longrightarrow \tilde{\rho} = \frac{\tilde{q}\tilde{n}_0}{ppc} = \frac{\frac{q}{e}\frac{n_0}{n_0}}{ppc} = \frac{\rho}{e n_0}$$

Gauss's law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \to dE = \frac{\rho}{\epsilon_0} dx \longrightarrow d\tilde{E} = \frac{\tilde{\rho}}{\tilde{\epsilon}_0} d\tilde{x} = \frac{\frac{\rho}{e n_0}}{\frac{\epsilon_0}{\epsilon_0}} \frac{dx}{\lambda_D} = \frac{dE}{\frac{\lambda_D e n_0}{\epsilon_0}}$$

Poisson equation:

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \to d\phi = -\frac{\rho}{\epsilon_0} dx^2 \longrightarrow d\tilde{\phi} = -\frac{\tilde{\rho}}{\tilde{\epsilon}_0} d\tilde{x}^2 = -\frac{\frac{\rho}{e n_0}}{\frac{\epsilon_0}{\epsilon_0}} \frac{dx^2}{\lambda_D^2} = \frac{d\phi}{\frac{\lambda_D^2 e n_0}{\epsilon_0}}$$

Field energy and kinetic energy:

$$U = \int \frac{1}{2} \epsilon_0 E^2 dx$$

$$K = \sum_i \frac{1}{2} m v_i^2 * w$$

$$\longrightarrow$$

$$\tilde{U} = \int \frac{1}{2} \tilde{\epsilon}_0 \tilde{E}^2 d\tilde{x} = \int \frac{1}{2} \frac{\epsilon_0}{\epsilon_0} \frac{E^2}{\left(\frac{\lambda_D e n_0}{\epsilon_0}\right)^2} \frac{dx}{\lambda_D} = \frac{U}{\frac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}}$$

$$\tilde{K} = \sum_i \frac{1}{2} \tilde{m} \tilde{v}_i^2 * \tilde{w} = \sum_i \frac{1}{2} \frac{m}{m_e} \left(\frac{v}{\lambda_D \omega_p}\right)^2 * \frac{w}{n_0 \lambda_D} = \frac{K}{\frac{\lambda_D^3 \omega_p^4 m_e^2 \epsilon_0}{e^2}}$$

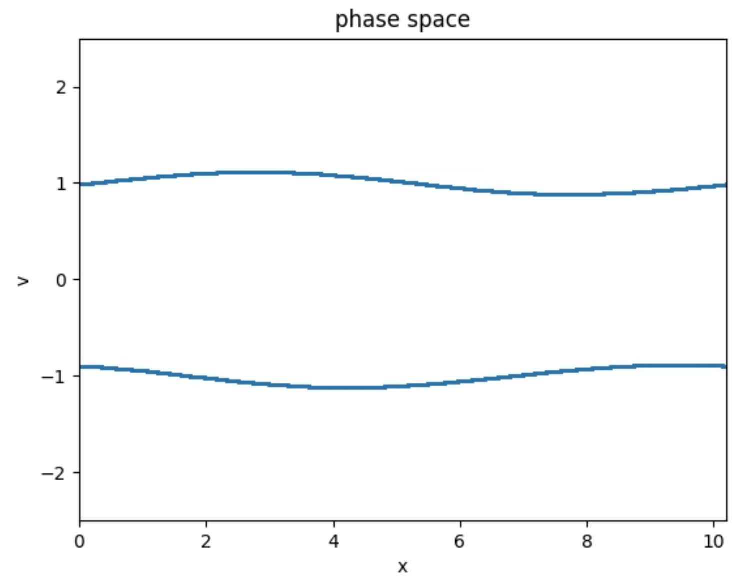
Momentum:

$$p = \sum_i m v_i * w \longrightarrow \tilde{p} = \sum_i \tilde{m} \tilde{v}_i * \tilde{w} = \sum_i \frac{m}{m_e} \frac{v}{\lambda_D \omega_p} * \frac{w}{n_0 \lambda_D} = \frac{p}{\frac{\lambda_D^2 \omega_p^3 m_e^2 \epsilon_0}{e^2}}$$

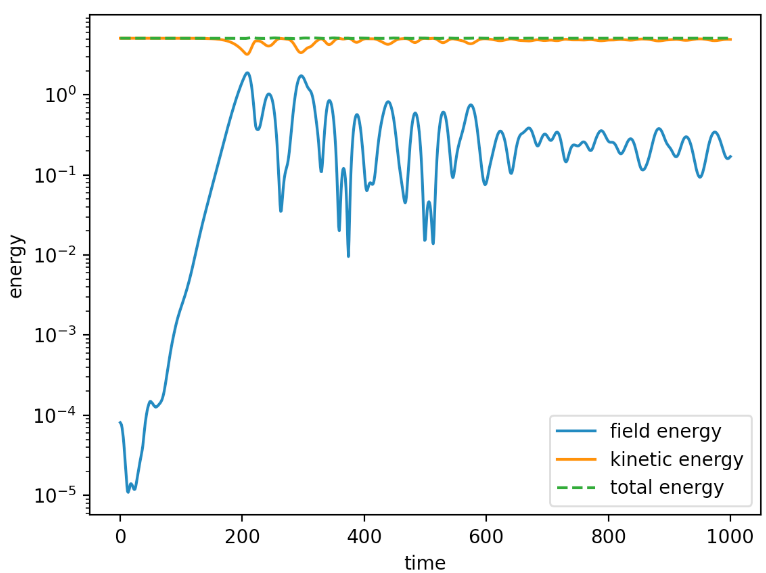
PIC

```python
# initial condition
GN = int(2 * np.pi /0.6124/0.1)
ppc = 5000
PN = ppc * GN
dx = 0.1 / Debye_length
dt = 0.1 * omega_D
x_electron = GN * dx * np.random.rand(PN)
t_max = 1000 * dt
```

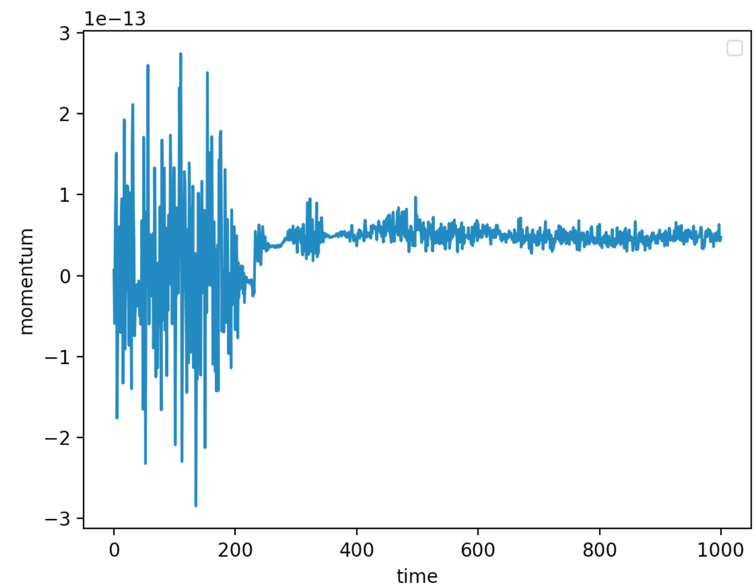
```python
v_electron = np.zeros(PN)
for i in range(PN):
    if i < 0.5 * PN:
        v_electron[i] = 1 / v_nor
    else:
        v_electron[i] = -1 / v_nor
```



Energy conservation



Momentum conservation



Normalize dimension: $\tilde{t} = \omega_p t, \tilde{x} = \frac{x}{\lambda_D}$……